

Additive modeling for location, scale, and shape parameters of the skew normal distribution

Modellizzazione additiva dei parametri di posizione, scala e forma della distribuzione normale asimmetrica

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Riassunto: In questo lavoro, si propone di modellare additivamente i parametri di locazione, scala e simmetria della normale asimmetrica (Azzalini (2005)), estendendo l'approccio di Rigby and Stasinopoulos (2005) a tale classe di distribuzioni. Il comportamento del modello è stato dapprima testato empiricamente mediante simulazioni Monte Carlo, verificando la robustezza dell'algoritmo di ottimizzazione utilizzato (differential evolution) e la stabilità nella stima dei parametri. Infine, tale modello è stato utilizzato per analizzare dati relativi all'evoluzione temporale della capacità cranica umana.

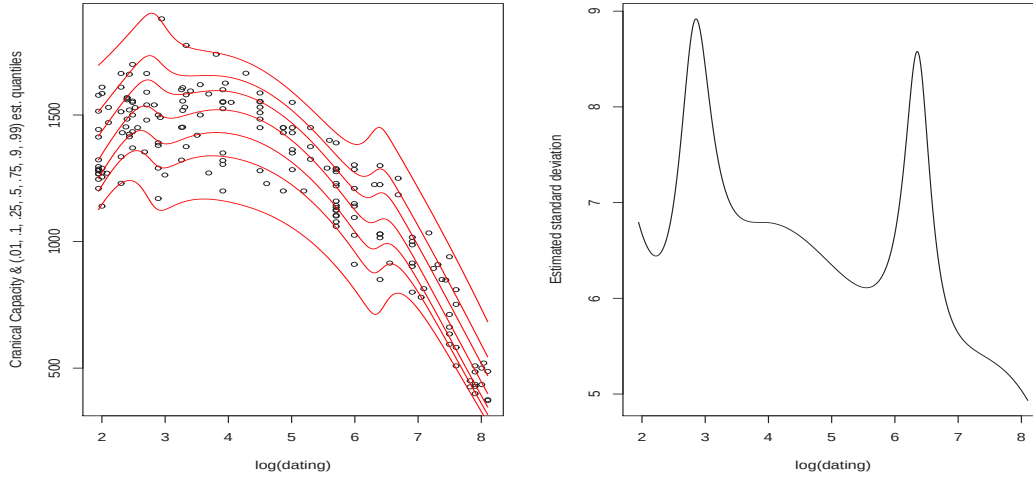
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1. Introduction

Generalized Additive Models (GAM) are flexible statistical methods used to identify and characterise nonlinear regression effects. In GAM the expected value of the real-valued dependent variable Y given the real-valued covariates X is modelled with an additive function. Extending the approach of Rigby and Stasinopoulos (2005), we propose to model the location, scale and shape explicitly as smooth functions of the explanatory variables by considering the class of the skew-normal distribution (Azzalini (2005)). The target is to model the IID conditional distribution of Y_i given X_i with a skew normal distribution, i.e. $(Y_i|X_i = x_i) \sim SN(\xi(x_i), \omega(x_i), \alpha(x_i))$, with location parameter $\xi(x_i) = B(x_i)' \beta_\xi$, scale parameter $\omega(x_i) = B(x_i)' \beta_\omega$, and shape parameter $\alpha(x_i) = B(x_i)' \beta_\alpha$, where $B(x_i) = (B_1(x_i), \dots, B_M(x_i))' \in \mathbb{R}^M$ is a fixed B-spline basis function of a M -dimensional function space, $M \in \mathbb{Z}^+$ and $\beta = (\beta_\xi, \beta_\omega, \beta_\alpha) \in \mathbb{R}^{3M}$ is the vector of real-valued parameters to be estimated. Parameters are estimated directly via maximum likelihood using differential evolution (Storn and Price (1997)), a stochastic search heuristic which is simple to implement, requires little or no parameter tuning, and is known for remarkable performance. The analysis of data about the evolution of human cranial capacity shows the relevance of the proposed model in analyzing real-world data.

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Figure 1: Cranial Capacity Evolution Data



2. Empirical Results

The empirical results are based on using a stochastic search heuristic, namely differential evolution, to estimate directly the parameters of the skew normal distribution. Differential Evolution can tackle optimisation problems as they are without requiring rigid properties, such as continuity, linearity or convexity of objective functions and constraints (Storn and Price (1997)). In this context, differential evolution can be a better alternative to classical optimisation algorithm in dealing with inflection points and the irregular shape of the likelihood function (Azzalini (2005)). In order to assess the robustness of differential evolution, we compare its performance on randomly generated data from univariate, multivariate skew-normal distribution and simulated additive models for location, scale and shape with skew normal distribution. The empirical evidence shows that DE converges steadily and quickly towards the real parameters values, even when the sample size is small and the parameter space is big.

Finally, we considered 181 observations of cranial capacity in cm^3 of hominid (Y) dated in thousand of years B.P. (X). We run 30 simulations for 3 and 4 splines. The three splines model, reported in figure 1 (left plot: scatter of the observed data and estimated quantiles, right plot: estimated standard deviation), allows capturing two evolutionary jumps (right plot), well known to anthropologists, and the variability in shape, location and scale of the data (left plot).

References

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